Descriptor revision

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Outline

Problems and limitations of AGM

Descriptor revision

The recovery postulate 1

$$K \subseteq (K \div p) + p \text{ (recovery)}$$

The recovery postulate 2

$$K$$

$s \in K$, $c \in K$

$$\downarrow$$

$$K \div c$$

$s \notin K$, $c \notin K$

$$\downarrow$$

$$K \div c + c$$

$s \notin K$, $c \in K$

This contradicts: $K \subseteq (K \div p) + p$ (recovery)

s = Cleopatra had a son, c = Cleopatra had a child

Explosion into infinity

If K is finite-based, then so is $K \div p$ (finite-based outcome)

(K is finite-based iff there is some finite set X with $K = \mathrm{Cn}(X)$.)

Problem: This postulate does not hold for partial meet contraction or for its transitively relational variant.

Pure contraction

$K \div p \subseteq K$ (inclusion)

Problem: Removal of a belief from the belief set always seems to depend on the acquisition of some new belief that is accepted. Therefore, pure contraction does not seem to be possible.

Pure expansion and the expansion property of revision 1

$K + p = \mathrm{Cn}(K \cup \{p\})$ (definition of expansion)

If $K \nvdash \neg p$ then $K * p = K + p = \mathrm{Cn}(K \cup \{p\})$
(expansion property of revision)

Pure expansion and the expansion property of revision 2

Counterexample: John is a neighbour about whom I initially know next to nothing.

Case 1: I am told that he goes home from work by taxi every day (t). This makes me believe that he is a rich man (r). Thus $r \in K * t$. Since $K \nvdash \neg t$, we have $K * t = K + t$, thus $t \rightarrow r \in K$.

Case 2: When told t, I am also told that John is a driver by profession (d). Since $K \nvdash \neg(t \& d)$ we have have $K * (t \& d) = K + (t \& d)$, thus $r \in K * (t \& d)$.

Impossibility of Ramsey test conditional 1

$p \rightarrowtail q \in K$ if and only if $q \in K * p$

(Ramsey test conditional)

Impossibility of Ramsey test conditional 2

The following conditions are incompatible:

- $p \rightarrowtail q \in K$ if and only if $q \in K * p$ (Ramsey test).
- $K * p = \mathrm{Cn}(K * p)$ (closure)
- $p \in K * p$ (success)
- If $p \nvdash \bot$ then $K * p \nvdash \bot$. (consistency)
- If $\neg p \notin K$ then $K \subseteq K * p$. (preservation)
- There are three sentences p, q, and r, and a belief set K such that $p \& q$, $p \& r$, and $q \& r$ are all inconsistent and that $\neg p \notin K$, $\neg q \notin K$, and $\neg r \notin K$. (non-triviality)

An underlying problem 1

- $\div$ Recovery
- $\div$ Explosion into infinity
- $\div$ Pure contraction
- $+$ Pure expansion
- $*$ Expansion property of revision
- $*$ Impossibility of Ramsey test conditional

All these problems are closely connected with the inordinate fine-grainedness of the AGM model.

An underlying problem 2

This is best seen on the *outcome level*.

The *outcome set* of $*$ is $\{X \mid (\exists p)(X = K * p\}$.

An underlying problem 3

The outcome set $\mathbb{K}$ of a transitively relational partial meet revision on K satisfies:

If $X \in \mathbb{K}$, then $X + p \in \mathbb{K}$

Counterexample: The above taxidriver example.

Cognitive inaccessibility

The selection function operates on cognitively inaccessible entities. If the language is logically infinite, then even if K is finite-based, $\gamma(K \perp p)$ denotes a selection among infinitely many non-finite-based objects.

The only road from one finite-based belief set to another is a detour into Cantor's paradise.

Outline

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Descriptor revision

Basic construction 1

Descriptor revision is based on two major principles, both of which are needed to obtain the advantages of this approach:

1: Selection among possible outcomes, not among possible worlds etc. The *outcome set* is taken for given. It consists of those belief sets that are sufficiently coherent and/or stable to be outcomes of a belief change.

Basic construction 2

2: A unified operator with a more general type of inputs, in the form of "success conditions" built on the metalinguistic belief operator $\mathfrak{B}$. Examples:

$K \circ \mathfrak{B}p$	Revision by p
$K \circ \neg\mathfrak{B}p$	Revocation ("contraction") by p
$K \circ \{\neg\mathfrak{B}p, \neg\mathfrak{B}q\}$	Multiple revocation ("contraction")
$K \circ \{\neg\mathfrak{B}p, \mathfrak{B}q\}$	Replacement
$K \circ (\mathfrak{B}p \vee \mathfrak{B}\neg p)$	Resolution (making up one's mind)

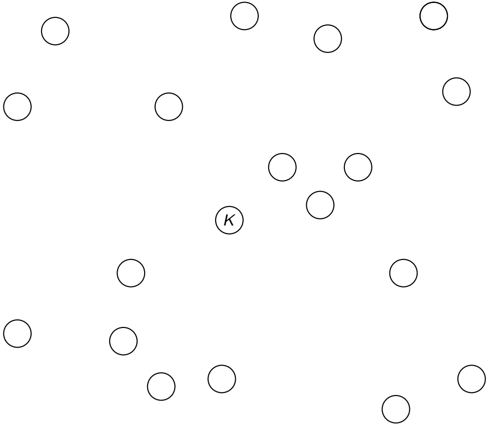
Etc.

Basic construction 3

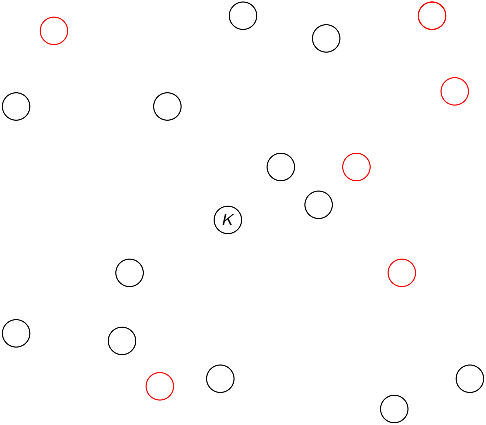
For any given general revision operator $\circ$ we can construct a long list of associated (and mutually connected) specialized operators, such as the sentential revision $K * p = K \circ \mathfrak{B}p$, etc.

One way to construct the general revision operator $\circ$ is to let $K \circ \Psi$ be closest element of the outcome set that satisfies Ψ. This requires an ordering or distance relation on belief sets (nota bene, not on the cognitively inaccessible sets referred to in other approaches).

Belief set semantics without possible worlds 1

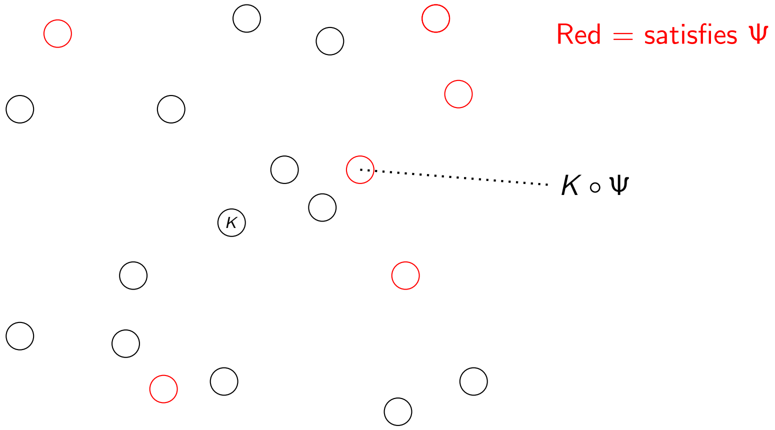


Belief set semantics without possible worlds 2



Red = satisfies Ψ

Belief set semantics without possible worlds 3



Belief set semantics without possible worlds 4

- Distances are assumed to be unique.
- Otherwise indeterministic belief change.

Axiomatic characterization of $\circ$

$K \circ \Psi = \mathrm{Cn}(K \circ \Psi)$ (closure)

$K \circ \Psi \Vdash \Psi$ or $K \circ \Psi = K$ (relative success)

If $K \circ \Xi \Vdash \Psi$ then $K \circ \Psi \Vdash \Psi$ (regularity)

If $K \Vdash \Psi$ then $K \circ \Psi = K$ (confirmation)

If $K \circ \Psi \Vdash \Xi$ then $K \circ \Psi = K \circ (\Psi \cup \Xi)$ (cumulativity)

Blockage relations 1

Let $\circ$ be a descriptor revision and $\mathbb{X}$ its outcome set. Its *blockage relation* is the relation $\rightharpoondown$ on $\mathbb{X}$ such that for all $X, Y \in \mathbb{X}$:

$X \rightharpoondown Y$ if and only if it holds for all Ψ that if $X \Vdash \Psi$, then $K \circ \Psi \neq Y$.

Blockage relations 2

An alternative axiomatic characterization of descriptor revision:
The outcome set $\mathbb{X}$ of $\circ$ is a set of belief sets, and its blockage relation $\rightharpoondown$ satisfies:

- transitivity,
- weak connectedness (If $X \neq Y$ then $X \rightharpoondown Y$ or $Y \rightharpoondown X$),
- asymmetry, and
- stability (If $X \neq K$ then $K \rightharpoondown X$).

$K \circ \Psi$ is is the unique $\rightharpoondown$-unblocked element among the set of Ψ-satisfying elements of $\mathbb{X}$, unless that set is empty, in which case $K \circ \Psi = K$.

Relations of Epistemic Proximity 1

- A generalization of entrenchment.
- Applies to descriptors rather than to sentences to be removed.
- Intuitively: $\Psi \geq \Xi$ (Ψ is at least as epistemically proximate as Ξ) if and only if the change in the belief system required to obtain assent to Ψ is not larger (more radical or far-reaching) than that required to obtain assent to Ξ.
- Semantically: $\Psi \geq \Xi$ if and only if the distance from K to the closest Ψ-satisfying potential outcome is not longer than that to the closest Ξ-satisfying potential outcome
- The symmetric part is denoted $\simeq$.

Relations of Epistemic Proximity 2

Postulates for Epistemic Promimity:

- Transitivity (If $\Psi \geq \Xi$ and $\Xi \geq \Sigma$, then $\Psi \geq \Sigma$),
- Counter-dominance (If $\Psi \Vdash \Xi$ then $\Xi \geq \Psi$),
- Coupling (If $\Psi \simeq \Xi$ then $\Psi \simeq \Psi \cup \Xi$),
- Amplification (Either $\Psi \cup \{\mathfrak{B}p\} \geq \Psi$ or $\Psi \cup \{\neg\mathfrak{B}p\} \geq \Psi$), and
- Absurdity avoidance ($\Psi \geq \perp$)

Relations of Epistemic Proximity 3

Constructing descriptor revision from a proximity relation:

$q \in K \circ \Psi$ if and only if either
(i) $\Psi \cup \{\mathfrak{B}q\} \simeq \Psi \succ \perp$ or
(ii) $q \in K$ and $\Psi \simeq \perp$.

Relations of Epistemic Proximity 4

Restriction of proximity relations to sentential revision:

The restriction to descriptors of the form $\mathfrak{B}p$ gives rise to a *believability relation* for sentential revision.

Relations of Epistemic Proximity 5

Restriction of proximity relations to contraction:

The restriction to descriptors of the form $\neg\mathfrak{B}p$ gives rise to standard *entrenchment relation*, satisfying the usual conditions:

- Transitivity (If $p \leq q$ and $q \leq r$, then $p \leq r$.)
- Dominance (If $p \vdash q$, then $p \leq q$)
- Conjunctiveness (Either $p \leq p\&q$ or $q \leq p\&q$.)
- Minimality ($p \notin K$ if and only if $p \leq q$ for all q.)
- Maximality (If $q \leq p$ for all q, then $\vdash p$.)

Iterated descriptor revision 1

Iterated revision with distance semantics:

With (not necessarily symmetric) pseudodistances there are no extra properties.

Symmetric distances make a difference.

Iterated descriptor revision 2

Iterated revision with symmetric distances axiomatized: The blockage relation is written $X \rightharpoondown_K Y$ and satisfies one more postulate:

- If $X_1 \not\rightharpoondown_{X_2} X_3, \;\; X_2 \not\rightharpoondown_{X_3} X_4, \dots \; X_{n-1} \not\rightharpoondown_{X_2} X_n$, then $X_1 \not\rightharpoondown_{X_2} X_n$. (negative transmission)

Iterated descriptor revision 3

None of the Darwiche-Pearl postulates for iterated revision is satisfied. This applies even if we require distances to be symmetric and one-dimensional:

- If $q \vdash p$, then $(X * p) * q = X * q$. (DP1)
- If $q \vdash \neg p$, then $(X * p) * q = X * q$. (DP2)
- If $X * q \vdash p$, then $(X * p) * q \vdash p$. (DP3)
- If $X * q \nvdash \neg p$, then $(X * p) * q \nvdash \neg p$ (DP4)

No wonder, since the DP postulates encode intuitions about orderings of possible worlds.

Descriptor conditionals 1

Introducing descriptor conditionals:

- Generalized descriptor conditionals of the form $\Psi \Rightarrow \Xi$.
- Can be interpreted with a generalized Ramsey test.
- Standard Ramsey conditionals: $p \rightarrowtail q$ iff $\mathfrak{B}p \Rightarrow \mathfrak{B}q$.
- Other variants such as $\neg\mathfrak{B}p \Rightarrow \neg\mathfrak{B}q$ and $\mathfrak{B}p \vee \mathfrak{B}\neg p \Rightarrow \mathfrak{B}p$.

Descriptor conditionals 2

Descriptor Ramsey test conditionals axiomatized:
(Restricted to expressions whose antecedents are satisfiable within $\mathbb{X}$.)

- If $\Psi \dashv\Vdash \Psi'$, then $\Psi \Rightarrow \Xi$ iff $\Psi' \Rightarrow \Xi$. (left logical equivalence)
- For all Ψ there is some $Y \subseteq \mathcal{L}$ such that for all Ξ:
  $\Psi \Rightarrow \Xi$ if and only if $Y \Vdash \Xi$ (unitarity)
- $\Psi \Rightarrow \Psi$ (reflexivity) and
- If $\Psi \Rightarrow \Xi$, then $\Psi \Rightarrow \Phi$ iff $\Psi \cup \Xi \Rightarrow \Phi$ (cumulativity)

Descriptor conditionals 3

Further developments with descriptor Ramsey test conditionals:

- Nested descriptor conditionals such as $\Psi \Rightarrow (\Phi \Rightarrow \Xi)$.
- Truth-functional combinations of descriptor conditionals (such as $\neg(\Psi \Rightarrow \Xi)$ and $(\Psi \Rightarrow \Xi_1) \vee (\Psi \Rightarrow \Xi_2)$).
- Revision by Ramsey test conditionals: $K \circ (\Psi \Rightarrow \Xi)$.

Autoepistemic beliefs

- Descriptors can be included in belief sets to express autoepistemic beliefs.
- Most plausible to include only some of the "true" descriptors.
- False autoepistemic beliefs can also be included.
- Dynamic autoepistemic beliefs can be expressed by including descriptor conditionals in the belief set. Gärdenfors's impossibility theorem does not apply!

Relations to AGM 1

- Sentential descriptor revision can be defined as $K * p = K \circ \mathfrak{B}p$.
- All full-blown AGM revisions (transitively relational partial meet revisions) are sentential descriptor revisions.

Relations to AGM 2

- Sentential descriptor revocation can be defined as $K \div p = K \circ \neg\mathfrak{B}p$.
- The simplest way to achieve a contraction operator is to assume that $\bigcup\mathbb{K} \subseteq K$.
- An AGM contraction (partial meet contraction) is a descriptor contraction if and only if it is a maxichoice transitively relational partial meet contraction.

Some advantages of descriptor revision

- A unified approach with generalized success conditions that allows for belief changes not accessible in the standard model.
- Solves the problems of cognitive inaccessibility and inordinate fine-grainedness and with them the most pressing problems of the standard model.
- Has resources for solving the pure contraction problem. (Pure contraction can be treated as an idealized version of revocation.)
- Iterated change obtainable with a plausible distance model.
- Allows the introduction of Ramsey test conditionals.
- Allows the introduction of autoepistemic beliefs.

Thanks for your attention!